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Estimation of Pulse Availability

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Operational availability, A_o, is an important consideration during the evaluation of system effectiveness and sustainability. A_o is commonly used and widely understood as a measure of steady-state system availability. However, a related term "pulse availability," A_P, has recently and more prominently been used in statements of requirements for some military systems under development. A_P is a subset of A_o and applies to a shorter and usually more intensive usage period, known as a "pulse." Various techniques are published on the estimation, calculation, and measurement of A_o; however, little if any can be found on the calculation and estimation of pulse availability, A_P. This article presents a straightforward approach for estimating A_P which will aid in the specification and evaluation of A_P requirements.

Key words: Availability, downtime, uptime, maintenance time, failure, time to restore, operational time.

Operational availability, A_o, is widely used as a readiness related objective in the specification of requirements for military systems. In general terms, A_o is the proportion of time a system is either operating or is capable of operating (called "uptime"), while being used in a specific manner in a typical maintenance and supply environment. In other words, A_o is the ratio of "*uptime*" to "*total time*." For complete definitions and discussion of A_o, see Pryor (2008).

Pulse availability, A_P, is a subset of operational availability that applies when the period of interest is not the steady-state availability, but system availability during a short, usually intensive period of operations.

Figure 1 shows a typical "failure–restore" cycle, which can be used to model steady-state operational availability. This cycle is theoretically repeated continuously because a system operates, experiences a failure, and accumulates downtime associated with maintenance and logistics delays until it is restored to an operational state.

Availability must be looked at differently when the period of interest is not steady state, but a shorter period, such as a period of intensive usage. Pulse availability, A_P, is usually higher than the steady-state A_o for the same operating tempo. When a failure occurs during the pulse, some of the downtime associated with the restoration of the system may extend outside of the pulse and is not counted as downtime for the pulse. One such possible example is illustrated in *Figure 2*. The amount of any increase in A_P over A_o depends primarily on the length of the pulse, the expected downtime per failure, and the steady-state A_o.

Estimation of operational availability

Two related A_o equations were described in previous published work by the author (Pryor, 2008) and are shown here as Equations 1 and 2. The specific equation or methodology used to determine A_o is of no importance; but Equations 1 and 2 are shown as one possible source.

$$A_o = \frac{1 - \mathrm{OPR} \times \mathrm{CMR_{ESS}}}{1 + \mathrm{OPR}(\mathrm{MCMT} + \mathrm{ADLT})/\mathrm{MTBF}}, \qquad (1)$$

$$A_o = \frac{1}{1 + (\mathrm{MCMT} + \mathrm{ALDT})/\mathrm{MTBF} + \mathrm{CMR_{ESS}}} \qquad (2)$$

where

ALDT = administrative and logistics delay time (per failure) in hours.

$\mathrm{CMR_{ESS}}$ = clock hour maintenance ratio—essential (maintenance time not associated with a critical failure that causes downtime; expressed as maintenance clock hours per operating hour). Includes both essential scheduled (preventive) main tenance and unscheduled (corrective) maintenance.

MCMT = mean corrective maintenance time (per failure) in hours.

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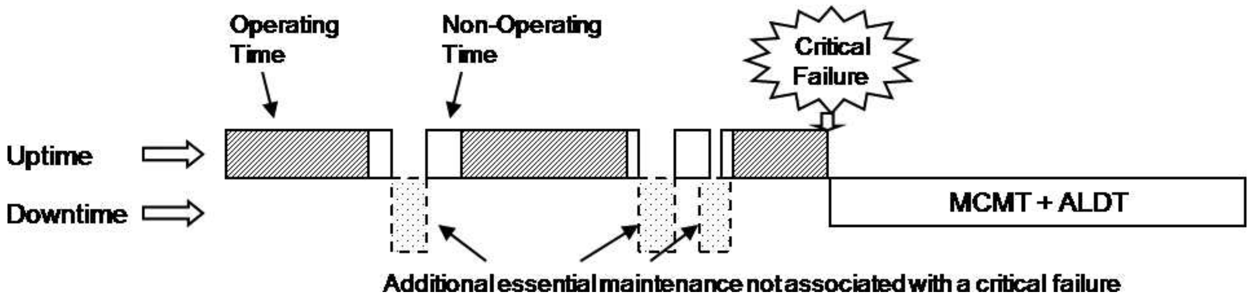

Figure 1. Steady-state failure-restore cycle.

MTBF = mean time between failure (representing "critical" failures that cause the system to be nonoperational) in hours.

OPR = operating rate (ratio of operating time to total time; i.e., a system that operates 12 hours per day would have an OPR of 50 percent).

If $CMR_{ESS} - [(1 - OPR)/OPR] \leq 0$, then Equation 1 can be used; otherwise use Equation 2. (Equation 2 is used for continuously operating or high op-tempo systems.)

Estimation of pulse availability

A_P will always be greater than or equal to the steady-state A_o because some of the downtime that was induced during the pulse will extend outside of the pulse, and therefore is not counted against the pulse A_o. The difference between A_P and steady-state A_o varies depending on the reliability relative to the pulse; the relative duration of the downtime; and, of course, the planned usage during the pulse.

If the MTBF is significantly greater than the length of the pulse, there is a high chance of completing the pulse without a failure. Then, a failure will occur only during a small percentage of pulses, and only that small percentage of pulses will experience any downtime during the pulse. For this case there will not be much difference between the A_P and the steady-state A_o.

If the MTBF is such that there is a good chance of experiencing one or more failures during the pulse, and if the average downtime is also high (relative to the pulse), then a significant portion of downtime can be expected to extend beyond the pulse, and the A_P will differ significantly from the steady-state A_o.

As shown in *Figure 3*, for situations where many failure–restore cycles occur during the pulse (i.e., very long pulses *or* low MTBF combined with low downtimes), then many failure–restore cycles occur during the pulse. In that case, because there are multiple failures and repairs during the pulse, most of the associated downtime occurs during the pulse and only downtime from the last failure can extend beyond the pulse. In this case, there will some but perhaps not a significant difference between the A_P and steady-state A_o.

We begin by modifying the normal A_o equation to include only the uptime and downtime, which occurs during the pulse, as shown in Equation 3.

$$A_P = \frac{\text{Uptime}_P}{\text{Uptime}_P + \text{Downtime}_P}, \tag{3}$$

where Uptime_P (UT_P) is the uptime within the pulse; and Downtime_P (DT_P) is the downtime within the pulse. As we have discussed previously, the difference between A_o

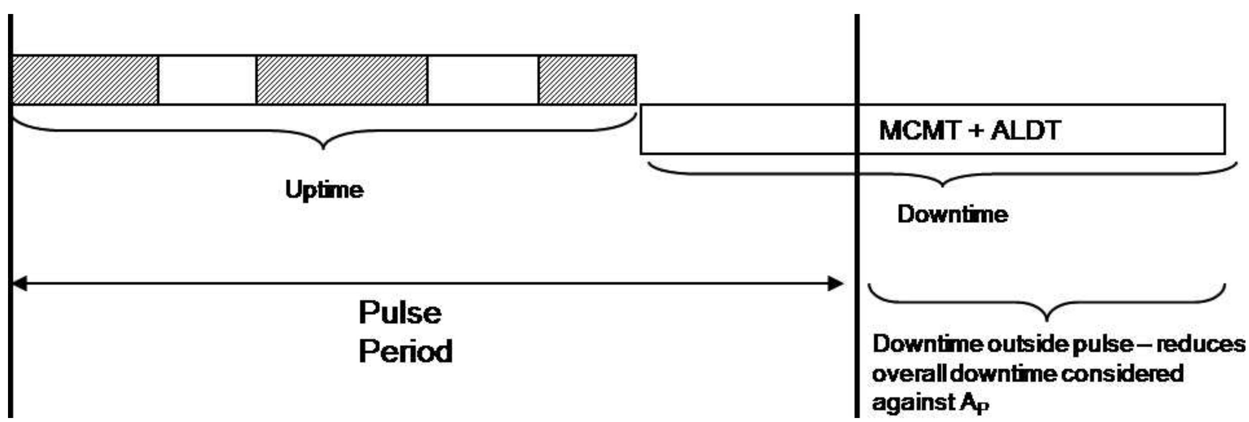

Figure 2. Uptime and downtime for pulse availability calculation.

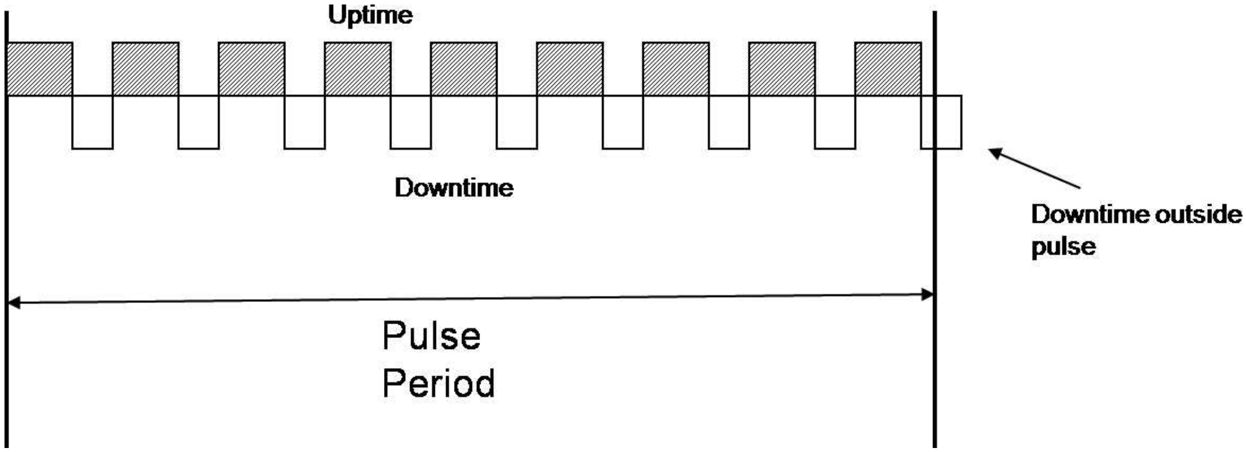

Figure 3. Pulse with many failure–restore cycles.

and A_P lies in the fact that some amount of the expected downtime extends beyond the pulse. The downtime that extends beyond the pulse is designated as DT_{OP}.

In general, the percentage of time we can expect to be operational is A_o, and the amount of time we can expect to be operational during the pulse, $UT_P = A_o \times PT$. Similarly, the amount of downtime we can expect during a pulse is $DT_P = (1 - A_o) \times PT$, less some amount of downtime normally expected that falls outside the pulse (DT_{OP}).

If a system is in a failed state at the end of a pulse, we will assume that there is an equal chance of being at any point in the restore process. So, given that the system is in a failed state, the average amount of downtime extending beyond the end of the pulse is one-half of the restore time, or (MCMT + ALDT)/2. By definition, $1 - A_o$ is the probability that the system is nonoperational at any given time, including the end of the pulse. So, there is a $1 - A_o$ probability that an (ALDT + MCMT)/2 amount of downtime will fall outside the pulse. This means that

$$\begin{aligned} DT_P &= \text{estimated downtime for pulse less expected downtime outside of pulse} \\ &= [(1-A_o)\times PT] \\ &\quad - [(1-A_o)(MCMT+ALDT)/2]. \end{aligned}$$

There is however one limitation arising from our assumption with respect to the average amount of downtime extending beyond the pulse. If the time to restore (TTR) is large with respect to the pulse time and we end the pulse in a failed state, then the system is much more likely to be early in the restore process and our assumption that one-half of the restore time extends beyond the pulse is not valid. Take for example a pulse time of 72 hours and a TTR of 200 hours. Using the previous assumption, we would effectively be reducing the pulse downtime by one-half of 200 hours, or 100 hours—longer than the actual pulse itself! To overcome this anomaly, we limit the time to restore to no more than the pulse time itself. Simulation results have borne out that no matter how high the restore time, once it exceeds the pulse time the resultant A_P is not affected. In that case, once a failure occurs, the failed system will not be returned to operation during the pulse. Now we can substitute into Equation 3:

$$\begin{aligned} A_P &= \frac{UT_P}{UT_P + DT_P} \\ &= \frac{A_o \times PT}{A_o \times PT + (1-A_o)PT - (1-A_o)(MCMT+ALDT)/2}; \end{aligned}$$

dividing numerator and denominator by PT and simplifying,

$$A_P = \frac{A_o}{A_o + (1-A_o) - [(1-A_o)(MCMT+ALDT)/2]/PT},$$

$$A_P = \frac{A_o}{1 - [(1-A_o)\times TTR/2PT]}, \tag{4}$$

where PT = pulse time period (hours),

$$\begin{aligned} TTR &= MCMT + ALDT \quad \text{if } (MCMT + ALDT) \le PT \quad \text{or} \\ TTR &= PT \quad \text{if } (MCMT + ALDT) > PT \end{aligned}$$

Note that Equation 4 is valid only when $A_o > 0.50$.

Verifying the output by comparing to Monte Carlo simulation results, it turns out that Equation 4 provides a good estimate of A_P except in cases of very low A_o. When A_o is less than 50 percent, Equation 4 begins to deviate significantly from simulated results. However, such low values of A_o are generally not acceptable and therefore not applied in real world situations.

The previous discussion relating to instances of long TTR leads to another simple methodology for estimating A_P in cases where the average time to restore exceeds the pulse time. If, as in the case of long TTR, we know

Table 1. Comparison of Equation 4 output to simulation results.

			A_P (Equation 4)					
			PT = 7,200 h		PT = 720 h		PT = 72 h	
MTBF (h)	**OPR**	**A_o (Equation 1)**	**Calc.**	**Sim.**	**Calc.**	**Sim.**	**Calc.**	**Sim.**
100	10/24	0.745	0.746	0.746	0.756	0.757	0.872	0.857
100	20/24	0.594	0.595	0.595	0.608	0.607	0.773	0.752
500	10/24	0.936	0.936	0.936	0.939	0.940	0.971	0.968
500	20/24	0.880	0.880	0.880	0.886	0.886	0.944	0.941

that systems will not be returned to operation during the pulse, we can use reliability, the probability of completing the pulse without a failure, to estimate the A_P.

Estimation of pulse availability for long times to restore

As stated previously, when the average time to restore is longer than the pulse time, a failed system will (on average) not be returned to operation during the pulse. Thus, we can compute the A_P using mission reliability as described further on. The exact reliability distribution used to calculate the pulse mission reliability does not matter; similar calculations can be performed for any distribution. But, for simplicity, we will use the exponential distribution.

We are interested in the probability of completing the pulse without a failure. This value will be designated as pulse reliability (R_P).

The derivation is straightforward and independent of the number of systems operating during the pulse, but it is more intuitive if we let N be the number of systems beginning the pulse.

For a group of N systems, UT_P is equal to the number of systems that make it through the pulse ($N \times R_P$) multiplied by the pulse time PT; summed with the number of systems that failed to make it through the pulse [$N(1 - R_P)$], multiplied by the uptime they accomplished prior to failure, UT_{FAIL}.

$$UT_P = [N \times R_P \times PT] + [N(1 - R_P) \times UT_{FAIL}]$$

UT_{FAIL} is easy to estimate for the exponential distribution because it assumes a constant failure or hazard rate, which means that failures are equally likely to occur at any time during the pulse. Given that a system has failed during the pulse, this lets us estimate the average time to failure as simply one-half of the PT. So, $UT_{FAIL} = PT/2$.

Now,

$$UT_P = (N \times R_P \times PT) + N(1 - R_P) \times PT/2,$$

Table 2. Comparison of Equation 4 and 5 to simulation results.

Avg. time to restore (TTR)	MTBF/OPR (h)	A_o (Equation 1)	A_P (Equation 4)	Sim. result	A_P (Equation 5)
25	12	0.324	0.345	0.344	0.500 (poor match)
25	60	0.706	0.724	0.726	0.545 (poor match)
25	120	0.828	0.840	0.841	0.651 (poor match)
25	600	0.960	0.963	0.963	0.893 (poor match)
82	12	0.128	0.170	0.163	0.500 (poor match)
82	60	0.423	0.506	0.496	0.545 (poor match)
82	120	0.594	0.672	0.658	0.651
82	600	0.880	0.911	0.909	0.893
162	12	0.069	0.129	0.083	0.500 (poor match)
162	24	0.129	0.229	0.168	0.501 (poor match)
162	48	0.229	0.372	0.314	0.525 (poor match)
162	72	0.308	0.471	0.438	0.568 (poor match)
162	96	0.372	0.542	0.543	0.612 (poor match)
162	120	0.426	0.597	0.582	0.651 (poor match)
162	180	0.526	0.690	0.685	0.725 (marginal)
162	240	0.597	0.748	0.748	0.774 (marginal)
162	360	0.690	0.816	0.819	0.835
162	480	0.748	0.856	0.869	0.870
162	600	0.787	0.881	0.887	0.893
162	1,200	0.881	0.937	0.942	0.943

Table 3.

Avg. time to restore (TTR)	MTBF/OPR (h)	Sim. result	A_P (Equation 5)
82	600	0.907	0.893
162	600	0.892	
242	600	0.887	
322	600	0.893	
402	600	0.891	

$$A_P = \text{Uptime}/\text{Totaltime},$$

$$A_P = \frac{(N \times R_P \times \text{PT}) + N(1 - R_P) \times \text{PT}/2}{N \times \text{PT}}.$$

Divide numerator and denominator by N and PT, and we get

$$A_P = R_P + (1 - R_P)/2. \tag{5}$$

Note that Equation 5 is valid for exponentially distributed failures; average time to restore greater than pulse time; and mean calendar time between failure greater than pulse time.

If one assumes that system failures follow the exponential distribution, Equation 5 can be used to quickly estimate pulse availability for systems with long times to restore (relative to the pulse time). Also, analysis of Monte Carlo simulation results indicates that Equation 5 provides a poor approximation when the mean calendar time between failure (MTBF/OPR) is less than PT.

Comparison of pulse availability estimations to simulation results

A rigorous proof of these formulas is not provided. However, it will be shown that they compare favorably to results of Monte Carlo simulations performed by the author. The author is confident that these results can be replicated by others willing to do so.

Table 1 shows results from the use of Equations 1 (A_o) and 4 (A_P) as compared with Monte Carlo simulation results for various inputs. The MTBFs vary between 100 hours and 500 hours; daily OT varies between 10 hours per day and 20 hours per day; pulse times vary between 72 hours, 720 hours, and 7,200 hours. For all cases, ALDT = 80 hours and MCMT = 2 hours. Each simulation result shown represents the average from a group of 10 systems operating for the specified pulse, repeated 1,000 times.

As can be seen, the A_P simulation results are within a few percentage points of the calculated values. Note that the simulated and calculated results from the higher MTBF are more comparable with each other. And, as expected, when the pulse time increases, the calculated A_P (Equation 4) approaches the steady-state A_o (Equation 1) and very closely matches the simulation results.

Table 2 compares the results of Equations 1, 4, and 5 with simulation results for a variety of cases.

It can be seen that Equation 5 does not match the simulated A_P for cases where either the TTR is less than the pulse time, or when the MTBF/OPR is less than the pulse time. *Table 3* shows simulation results for increasing TTR—showing as expected no affect on A_P when the TTR is increased well above the PT.

Conclusion

The equations and methodologies in this article describe an original approach to estimation of pulse availability. The outputs of Equations 4 and 5 closely match results of Monte Carlo simulations written to specifically measure A_P over typical operating cycles. The only limitations are that Equation 4 is valid only when the operational availability is above 0.50 (hardly an actual limitation). Equation 5 is valid when the failure distribution is exponential; and the reliability (MTBF) and average time to restore are both high relative to the pulse time. Although various techniques are available to predict A_o, very little can be found on estimation of A_P, and the author hopes to fill that void with a simple and straightforward approach. ❑

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